

Earthquakes clustering based on maximum likelihood estimation of point process conditional intensity function

Raggruppamento di eventi sismici basato sulla stima di massima verosimiglianza della funzione di intensità di un processo di punto

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Riassunto: In questo lavoro viene proposta una tecnica di clustering progettata per separare le due principali componenti dell'attività sismica di un'area sismogenetica: la sismicità di fondo e quella relativa a sequenze sismiche. Il metodo qui proposto assegna ciascun terremoto a una sequenza sismica o all'insieme degli eventi che costituiscono la sismicità di fondo, sulla base dei valori assunti dalla funzione di intensità del processo di punto generatore degli eventi osservati. I parametri che caratterizzano tale funzione sono stimati tramite il metodo della massima verosimiglianza. La partizione dell'insieme di eventi è modificata iterativamente in modo da incrementare la verosimiglianza globale. Questa tecnica impiega metodi di stima non parametrica in una procedura complessa di massimizzazione della funzione di verosimiglianza di un processo di punto.

Keywords: Earthquakes clustering, point process, intensity function, MLE.

1. Introduction

Because of the different seismogenic features controlling the kind of seismic release of background and clustered seismicity (Adelfio *et al*, 2005), to describe the seismicity of an area in space, time and magnitude domains, it is useful to study separately the features of *independent* events and strongly correlated ones. The two different kinds of sets of events give different information on the seismicity of an area. For the estimation of parameters of phenomenological laws useful for the description of seismicity, we need a good definition of the earthquake clusters. Furthermore the prediction of the occurrence of large earthquakes (related to the assessment of seismic risk in space and time) is complicated by the presence of clusters of aftershocks, that are superimposed to the background seismicity and shade its principal characteristics.

For this purposes the preliminary subdivision of a seismic catalog in background seismicity (represented by isolated events, that do not trigger any further event, and the mainshock of each seismic sequence) and clustered events is required. At this regard, a seismic sequences detection technique is presented; it is based on MLE of parameters that identify the conditional intensity function of a model describing seismic activity as a clustering-process and representing a slight modification of the ETAS model (Epidemic Type Aftershocks-Sequences model; Ogata, 1988, Ogata, 1998).

2. Conditional intensity function in point processes

The conditional intensity function of a space-time point process can be defined as:

$$\lambda(t, x, y|H_t) = \lim_{\Delta t, \Delta x, \Delta y \rightarrow 0} \frac{Pr_{\Delta t \Delta x \Delta y}(t, x, y|H_t)}{\Delta t \Delta x \Delta y} \quad (1)$$

where H_t is the space-time occurrence history of the process up to time t ; $\Delta t, \Delta x, \Delta y$ are time and space increments; $Pr_{\Delta t \Delta x \Delta y}(t, x, y|H_t)$ is the history-dependent probability that an event occurs in the volume $\{[t, t + \Delta t) \times [x, x + \Delta x) \times [y, y + \Delta y)\}$. The conditional intensity function completely identifies the features of the associated point process (i.e. if it is independent of the history but dependent only on the current time and the spatial locations (1) supplies a nonhomogeneous Poisson process; a constant conditional intensity provides a stationary Poisson process).

Wider classes of point processes are defined by different conditional intensity functions (Schoenberg and Bolt, 2000).

ETAS model is a self-exciting point process describing earthquakes catalogs as a realization of a branching or epidemic-type point process. In particular it could be considered as an extension of the Hawkes model (Hawkes, 1971), which is a generalized Poisson cluster process associating to cluster centers a branching process of descendants.

The conditional intensity function provides a quantitative evaluation of future seismic activity. It is proportional to the probability that an event with magnitude m will take place at time t , in the epicenter of coordinates (x, y) .

3. The proposed clustering method

The clustering technique that we propose leads to an intensive computational procedure, implemented by software R (R Development Core Team, 2005). It identifies a partition of a catalog of seismic events, $\mathcal{P}_{k+1}$, formed by $k + 1$ sets: the background seismicity and the clustered events (k disjoint sets). It iteratively changes the partition assigning events either to the background seismicity or to the j -th cluster ($j = 1, \dots, k$), on the basis of the likelihood function variation due to their moving from a set to another one.

Let $[T_0, T_{max}]$ and Ω_{xy} time and space domains of observation respectively; according to the theory of point process, the likelihood function to be maximized is:

$$\log L(\boldsymbol{\theta}) = \sum_{i=1}^n \log \lambda(x_i, y_i, t_i; \boldsymbol{\theta}) - \int_{T_0}^{T_{max}} \int_{\Omega_{xy}} \lambda(x, y, t; \boldsymbol{\theta}) dx dy dt \quad (2)$$

where:

$$\lambda(x, y, t; \boldsymbol{\theta}) = \lambda_t \mu(x, y) + K_0 \sum_{\substack{j=1 \\ (t_j < t)}}^k g_j(x, y) \frac{\exp[\alpha(m_j - m_0)]}{(t - t_j + c_j)^{p_j}} \quad (3)$$

and $\boldsymbol{\theta} = (\lambda_t, K_0, c_j, p_j, \alpha)$. In (3) t_j and m_j are time of the first event and magnitude of the mainshock of the cluster j , $g_j(x, y)$ is the space density of the cluster j and $\mu(x, y)$ is the background one; K_0 and λ_t are the weights of the clustered seismicity and of the background one, respectively. Background seismicity is assumed stationary in time, while

time aftershock activity is represented by a non stationary Poisson process according to the modified Omori formula (Utsu, 1961), of parameters c_j, p_j , relating the occurrence rate of aftershocks to the mainshock magnitude (with α measuring the influence on the relative weight of each sequence and m_0 the completeness threshold of magnitude, that is the lower bound for which earthquakes with higher values of magnitude are surely recorded in the catalog).

In our approach space density both of background seismicity and of each cluster, differently by ETAS model, is estimated by a bivariate kernel estimator: it is computed either using only the independent events (isolated and mainshocks) or points belonging to the cluster, including the mainshock, respectively. In both cases the smoothing constant is evaluated with Silverman's formula (Silverman, 1986).

In the evaluation of (3), different kinds of parametrization, that allow to take into account for different assumptions on the seismicity of an area, are considered (e.g. Omori's law parameters of the k clusters can be assumed equal or distinct in each cluster). The choices can be compared at the end of the procedure.

3.1. Main steps

To start, the proposed method needs an initial partition, found by a like single-linkage method procedure. In this first step, choosing a threshold parameter, an initial number of clusters is determined, but then the classification of the events can change, because each event could change its position moving to the set for which the likelihood function is maximized.

The space densities and the ML estimates of the parameters of the intensity function (3) are evaluated. Moving on, for each event U_i , ($i = 1, \dots, n$), isolated or belonging to the cluster s , we first find the cluster which maximizes the conditional intensity (3): r is the index corresponding to the maximum of the k contributions in the sum:

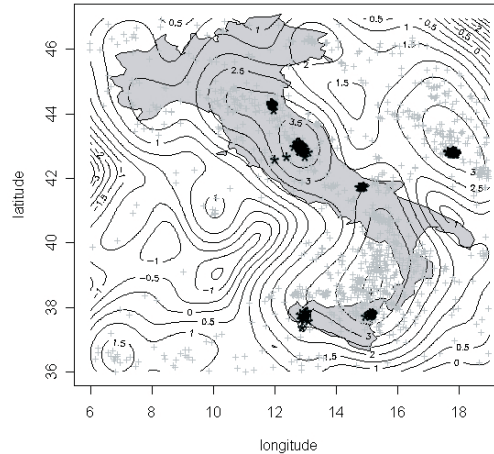
$$\sum_{\substack{j=1 \\ (t_j < t_i)}}^k g_j(x_i, y_i) \frac{\exp[\alpha(m_j - m_0)]}{(t_i - t_j + c)^p} \quad (4)$$

Then we try to move all events (one by one) from their current position (either from the cluster s to the cluster r or from the background to the cluster r or from the cluster r to the background) and we compute the change in (2) due to these movements, using the current value of the estimated parameters. If the current value of the likelihood increases the movements become effective. If at least one event changes its position the partition $\mathcal{P}_{k+1}$ is updated and the algorithm restarts from the intensity estimation step; the number of clusters could decrease during the iterative optimization. The iterative procedure stops when the current classification does not change after a whole iteration.

4. Conclusion

Although the proposed method has some critical aspects that have to be solved (as the computational burden and a certain dependence of the convergence of the iterative algorithm on some initial choices) it could be the basis to carry out an analysis of the complexity of the seismogenetic processes relative to each sequence and to the

Figure 1: Clusters (in black * symbol) and isolated events (in grey + symbol) found in the Italian catalog (1964 - 2002) and contour plot of the estimated space density



background seismicity, separately. It returns a plausible separation of the different components of seismicity and clusters that have a good interpretability, estimating the space pattern of the induced seismicity through non parametric methods, using only the events of each cluster. It also returns the value of the likelihood function and the estimate of the intensity function for each point (see for example Figure 1).

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